1. $\lim_{x\to a} [f(x) \pm g(x)] = \left(\lim_{x\to a} f(x)\right) \pm \left(\lim_{x\to a} g(x)\right)$

2. $\lim_{x\to a} [c f(x)] = c \left(\lim_{x\to a} f(x)\right)$

3. $\lim_{x\to a} [f(x) g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$

4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided $\lim_{x\to a} g(x) \neq 0$

5. $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$

6. $\lim_{x\to a} c = c$

7. $\lim_{x\to a} x = a$

8. $\lim_{x\to a} x^n = a^n$

9. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ (if n is even, assume $a > 0$)

10. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ (if n is even, assume $\lim_{x\to a} f(x) > 0$)

Ex: ⓐ $\lim_{x\to 3}(5x^3-3x^2+x-6)$

① $= \lim_{x\to3} 5x^3 - \lim_{x\to3} 3x^2 + \lim_{x\to3} x - \lim_{x\to3} 6$

② $= \underbrace{5\left(\lim_{x\to3} x^3\right)}_{⑧} - \underbrace{3\left(\lim_{x\to3} x^2\right)}_{⑧} + \underbrace{\lim_{x\to3} x}_{⑦} - \underbrace{\lim_{x\to3} 6}_{⑥}$

$= 5(3^3) - 3(3^2) + 3 - 6$

$= 5(27) - 27 + 3 - 6 = 105$

Direct Substitution Property (DSP)

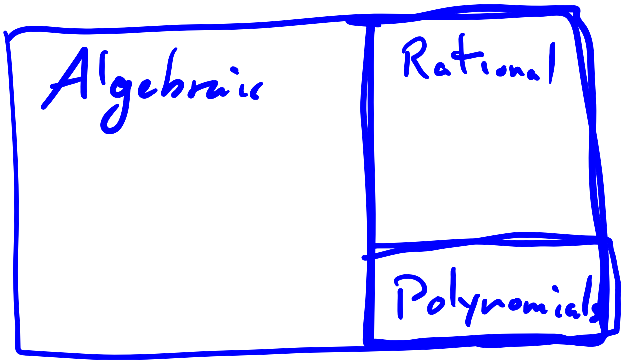
If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x) = f(a)$$

$$\frac{x^3+2x^2-1}{5-3x}$$

Domain: $x = \frac{5}{3}$

The DSP also applies to "algebraic functions"



- addition
- subtraction
- multiplication
- division
- exponentiation (includes roots)

$$\frac{\sqrt{3t^2-6}+t^2-1}{\sqrt[3]{t^7+2}-t^{3/2}}$$

Ex: $\displaystyle\lim_{x\to 2}\sqrt{\frac{2x^2+1}{3x-2}} \overset{DSP}{=} \sqrt{\frac{2(2)^2+1}{3\cdot 2-2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

Ex: $\displaystyle\lim_{x\to 4}\frac{x^2-16}{x-4} = \lim_{x\to 4}\frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}} = \lim_{x\to 4}(x+4) = \boxed{8}$

Ex: $\displaystyle\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$(a+b)(a-b)$
$= a^2-b^2$

$$= \lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

Function Replacement Rule

If $f(x) = g(x)$ when $x \neq a$, then

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$, provided the limit exists.